Basic equations

operators	$\widehat{\mathrm{x}} \Longrightarrow x$	$\widehat{\mathrm{p}}_x \Longrightarrow -\mathrm{i}\hbar\dfrac{\partial}{\partial x}$
Schrödinger's equation	$\mathrm{i}\hbar\dfrac{\partial\Psi}{\partial t} = \widehat{\mathrm{H}}\Psi$	$\mathrm{i}\hbar\dfrac{\partial\Psi}{\partial t} = -\dfrac{\hbar^2}{2m}\dfrac{\partial^2\Psi}{\partial x^2} + V(x)\Psi(x,t)$
stationary state	$\Psi_n(x,t) = \psi_n(x)\mathrm{e}^{-\mathrm{i}E_n t/\hbar}$	
time-independent S.E.	$\widehat{\mathrm{H}}\psi_n(x) = E_n\psi_n(x)$	$-\dfrac{\hbar^2}{2m}\dfrac{\mathrm{d}^2\psi_n}{\mathrm{d}x^2} + V(x)\psi_n(x) = E_n\psi_n(x)$
probability	$p_i = \left\lvert \int_{-\infty}^{\infty} \psi_i^*(x)\Psi(x,t)\mathrm{d}x \right\rvert^2$	probability density $= \lvert\Psi(x,t)\rvert^2$
expectation value	$\langle A\rangle = \sum_i p_i A_i$	$\langle A\rangle = \int_{-\infty}^{\infty} \Psi^*(x,t)\widehat{\mathrm{A}}\Psi(x,t)\,\mathrm{d}x$
uncertainty	$\Delta A = \left(\langle A^2\rangle - \langle A\rangle^2\right)^{1/2}$	$\Delta x\,\Delta p_x \geq \hbar/2$
Ehrenfest's theorem	$\dfrac{\mathrm{d}\langle x\rangle}{\mathrm{d}t} = \dfrac{\langle p_x\rangle}{m}$	$\dfrac{\mathrm{d}\langle p_x\rangle}{\mathrm{d}t} = -\left\langle \dfrac{\partial V}{\partial x}\right\rangle$
Boundary conditions	$\psi(x)$ finite at $\pm\infty$, cts everywhere	$\mathrm{d}\psi/\mathrm{d}x$ cts where $V(x)$ finite

One-dimensional infinite square well $(n = 1, 2, \ldots)$

(symmetric well)

$E_n = \dfrac{n^2\pi^2\hbar^2}{2mL^2}$ $\qquad \psi_n(x) = \sqrt{\dfrac{2}{L}}\sin\left(\dfrac{n\pi x}{L}\right) \quad (0 \leq x \leq L)$

$\psi_n(x) = \sqrt{\dfrac{2}{L}}\cos\left(\dfrac{n\pi x}{L}\right) \quad (n \text{ odd})$

$\psi_n(x) = \sqrt{\dfrac{2}{L}}\sin\left(\dfrac{n\pi x}{L}\right) \quad (n \text{ even})$

Harmonic oscillator $(n = 0, 1, 2, \ldots)$

$E_n = (n + \tfrac{1}{2})\hbar\omega_0$ $\qquad \omega_0 = \sqrt{C/m}$ $\qquad a = \sqrt{\hbar/m\omega_0}$ $\qquad \widehat{\mathrm{H}} = \left(\widehat{\mathrm{A}}^\dagger\,\widehat{\mathrm{A}} + \tfrac{1}{2}\right)\hbar\omega_0$

$\widehat{\mathrm{A}}^\dagger\psi_n(x) = \sqrt{n+1}\,\psi_{n+1}(x)$ $\qquad \widehat{\mathrm{A}}\,\psi_n(x) = \sqrt{n}\,\psi_{n-1}(x)$ $\qquad \widehat{\mathrm{A}}\,\psi_0(x) = 0$

$\widehat{\mathrm{x}} = \dfrac{a}{\sqrt{2}}\left(\widehat{\mathrm{A}} + \widehat{\mathrm{A}}^\dagger\right)$ $\qquad \widehat{\mathrm{p}}_x = \dfrac{-\mathrm{i}\hbar}{\sqrt{2}\,a}\left(\widehat{\mathrm{A}} - \widehat{\mathrm{A}}^\dagger\right)$ $\qquad \widehat{\mathrm{A}}\widehat{\mathrm{A}}^\dagger - \widehat{\mathrm{A}}^\dagger\widehat{\mathrm{A}} = 1$

Free particle, scattering and tunnelling

$\Psi(x,t) = \dfrac{1}{\sqrt{2\pi}}\displaystyle\int_{-\infty}^{\infty} A(k)\mathrm{e}^{\mathrm{i}(kx - E_k t/\hbar)}\,\mathrm{d}k$ $\qquad A(k) = \dfrac{1}{\sqrt{2\pi}}\displaystyle\int_{-\infty}^{\infty} \Psi(x,0)\mathrm{e}^{-\mathrm{i}kx}\,\mathrm{d}x$ $\qquad p_x = \hbar k$

$\displaystyle\int_{-\infty}^{\infty} \lvert A(k)\rvert^2\,\mathrm{d}k = 1$ $\qquad \langle p_x\rangle = \displaystyle\int_{-\infty}^{\infty} \hbar k\,\lvert A(k)\rvert^2\,\mathrm{d}k$

$j_x(x,t) = -\dfrac{\mathrm{i}\hbar}{2m}\left(\Psi^*\dfrac{\partial\Psi}{\partial x} - \Psi\dfrac{\partial\Psi^*}{\partial x}\right)$ $\qquad T = \dfrac{4k_1k_2}{(k_1+k_2)^2} \quad$ (step)

Complex numbers

$z = x + \mathrm{i}y = r\mathrm{e}^{\mathrm{i}\theta}$	$z^* = x - \mathrm{i}y = r\mathrm{e}^{-\mathrm{i}\theta}$	$\lvert z\rvert^2 = zz^* = x^2 + y^2 = r^2$
$\mathrm{Re}(z) = \dfrac{z + z^*}{2}$	$\mathrm{Im}(z) = \dfrac{z - z^*}{2\mathrm{i}}$	$z^n = r^n \mathrm{e}^{\mathrm{i}n\theta}$
$\mathrm{e}^{\mathrm{i}\theta} = \cos\theta + \mathrm{i}\sin\theta$	$\cos\theta = \dfrac{\mathrm{e}^{\mathrm{i}\theta} + \mathrm{e}^{-\mathrm{i}\theta}}{2}$	$\sin\theta = \dfrac{\mathrm{e}^{\mathrm{i}\theta} - \mathrm{e}^{-\mathrm{i}\theta}}{2\mathrm{i}}$
$\mathrm{e}^{\pm\mathrm{i}\pi} = -1$	$\mathrm{e}^{\mathrm{i}\pi/2} = \mathrm{i}$	$\mathrm{e}^{-\mathrm{i}\pi/2} = -\mathrm{i}$

Elementary functions $(a > 0, b > 0)$

$\mathrm{e}^x\mathrm{e}^y = \mathrm{e}^{x+y}$	$\ln a + \ln b = \ln(ab)$	$\mathrm{e}^{\ln a} = \ln(\mathrm{e}^a) = a$
$\mathrm{e}^x = \cosh x + \sinh x$	$\cosh x = \dfrac{\mathrm{e}^x + \mathrm{e}^{-x}}{2}$	$\sinh x = \dfrac{\mathrm{e}^x - \mathrm{e}^{-x}}{2}$
$\cos(\theta \pm \pi) = -\cos\theta$	$\sin(\theta \pm \pi) = -\sin\theta$	$\tan(\theta \pm \pi) = \tan\theta$
$\cos(\theta + \pi/2) = -\sin\theta$	$\sin(\theta + \pi/2) = \cos\theta$	$\tan(\theta + \pi/2) = -\cot\theta$
$\cos(\theta - \pi/2) = \sin\theta$	$\sin(\theta - \pi/2) = -\cos\theta$	$\tan(\theta - \pi/2) = -\cot\theta$
$\cos(2\theta) = \cos^2\theta - \sin^2\theta$	$\sin(2\theta) = 2\sin\theta\cos\theta$	$\tan(2\theta) = 2\tan\theta/(1 - \tan^2\theta)$

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$	$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$
$\sin A\sin B = \frac{1}{2}\big(\cos(A - B) - \cos(A + B)\big)$	$\cos A\cos B = \frac{1}{2}\big(\cos(A - B) + \cos(A + B)\big)$
$\sin A\cos B = \frac{1}{2}\big(\sin(A - B) + \sin(A + B)\big)$	$\cos^2 A + \sin^2 A = 1$

Physical constants

Planck's constant	h	$6.63 \times 10^{-34}\,\mathrm{J\,s}$	Planck's constant/2π	$\hbar$	$1.06 \times 10^{-34}\,\mathrm{J\,s}$
vacuum speed of light	c	$3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$	Coulomb law constant	$\frac{1}{4\pi\varepsilon_0}$	$8.99 \times 10^{9}\,\mathrm{m\,F^{-1}}$
permittivity of free space	ε_0	$8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$	permeability of free space	μ_0	$4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}$
Boltzmann's constant	k	$1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$	Avogadro's constant	N_m	$6.02 \times 10^{23}\,\mathrm{mol^{-1}}$
electron charge	$-e$	$-1.60 \times 10^{-19}\,\mathrm{C}$	proton charge	e	$1.60 \times 10^{-19}\,\mathrm{C}$
electron mass	m_e	$9.11 \times 10^{-31}\,\mathrm{kg}$	proton mass	m_p	$1.67 \times 10^{-27}\,\mathrm{kg}$
Bohr radius	a_0	$5.29 \times 10^{-11}\,\mathrm{m}$	atomic mass unit	u	$1.66 \times 10^{-27}\,\mathrm{kg}$

Definite integrals for positive integers n and m

$$\int_{-a}^{a} f(x)\,\mathrm{d}x = 0 \qquad (f(x) \text{ an odd function})$$

$$\int_{-a}^{a} f(x)\,\mathrm{d}x = 2\int_{0}^{a} f(x)\,\mathrm{d}x \qquad (f(x) \text{ an even function})$$

$$\int_{0}^{\pi} \sin(nx)\sin(mx)\,\mathrm{d}x = \frac{\pi}{2}\,\delta_{nm}$$

$$\int_{0}^{\pi} \cos(nx)\cos(mx)\,\mathrm{d}x = \frac{\pi}{2}\,\delta_{nm}$$

$$\int_{-\pi/2}^{\pi/2} \sin(nx)\sin(mx)\,\mathrm{d}x = \frac{\pi}{2}\,\delta_{nm} \qquad (n+m \text{ even})$$

$$\int_{-\pi/2}^{\pi/2} \cos(nx)\cos(mx)\,\mathrm{d}x = \frac{\pi}{2}\,\delta_{nm} \qquad (n+m \text{ even})$$

$$\int_{0}^{n\pi} \cos^2 x\,\mathrm{d}x = \frac{n\pi}{2}$$

$$\int_{0}^{n\pi} \sin^2 x\,\mathrm{d}x = \frac{n\pi}{2}$$

$$\int_{0}^{n\pi} x\cos^2 x\,\mathrm{d}x = \frac{n^2\pi^2}{4}$$

$$\int_{0}^{n\pi} x\sin^2 x\,\mathrm{d}x = \frac{n^2\pi^2}{4}$$

$$\int_{0}^{n\pi} x^2\cos^2 x\,\mathrm{d}x = \frac{n^3\pi^3}{6} + \frac{n\pi}{4}$$

$$\int_{0}^{n\pi} x^2\sin^2 x\,\mathrm{d}x = \frac{n^3\pi^3}{6} - \frac{n\pi}{4}$$

$$\int_{-n\pi/2}^{n\pi/2} \cos^2 x\,\mathrm{d}x = \frac{n\pi}{2}$$

$$\int_{-n\pi/2}^{n\pi/2} \sin^2 x\,\mathrm{d}x = \frac{n\pi}{2}$$

$$\int_{-n\pi/2}^{n\pi/2} x^2\cos^2 x\,\mathrm{d}x = \frac{n^3\pi^3}{24} + \frac{n\pi}{4}(-1)^n$$

$$\int_{-n\pi/2}^{n\pi/2} x^2\sin^2 x\,\mathrm{d}x = \frac{n^3\pi^3}{24} - \frac{n\pi}{4}(-1)^n$$

$$\int_{-n\pi/2}^{n\pi/2} x^2\cos x\,\mathrm{d}x = (-1)^{(n+3)/2}\left(\frac{n^2\pi^2}{2} - 4\right), \quad (n \text{ odd})$$

$$\int_{-n\pi/2}^{n\pi/2} x^2\cos x\,\mathrm{d}x = (-1)^{n/2}\,2\pi n, \qquad (n \text{ even})$$

$$\int_{-\infty}^{\infty} \mathrm{e}^{-x^2}\mathrm{d}x = \sqrt{\pi}$$

$$\int_{0}^{\infty} x^n\mathrm{e}^{-x}\mathrm{d}x = n!$$

$$\int_{0}^{\infty} x^{2n+1}\mathrm{e}^{-x^2}\mathrm{d}x = \frac{n!}{2} \qquad (n \geq 0)$$

$$\int_{-\infty}^{\infty} \mathrm{e}^{-x^2}\mathrm{e}^{\mathrm{i}kx}\,\mathrm{d}x = \sqrt{\pi}\,\mathrm{e}^{-k^2/4} \qquad (k \text{ real})$$

$$\int_{-\infty}^{\infty} x^{2n}\mathrm{e}^{-x^2}\mathrm{d}x = \frac{1\times 3\times\cdots\times(2n-1)}{2^n}\sqrt{\pi} \qquad (n \geq 1)$$

$$n! = 1\times 2\times\cdots\times n \qquad 0! = 1$$